Mixed oligopoly and spatial price discrimination with foreign firms

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ABSTRACT

This paper is the first to examine the welfare consequences of foreign competition in a mixed oligopoly set in a linear model of spatial price discrimination. It demonstrates that the entry of a foreign firm often lowers domestic welfare. This results because the public firm locates largely independently of the presence of the foreign firm and because the profit earned by the foreign firm reduces domestic welfare. Privatization of the public firm typically lowers domestic welfare but can increase global welfare. Thus, domestic governments are unlikely to allow foreign entry and when they do, they are unlikely to privatize the public firm despite the potential rise in global welfare.

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1. Introduction

The idea that a public firm can regulate oligopolistic behavior dates from at least Merrill and Schneider (1966) and researchers since then have spent considerable effort determining whether or not the presence of the public firm actually improves welfare. Despite the fact that the public firm maximizes social welfare, because of its interaction with private profit-maximizing firms its presence does not routinely increase welfare. Thus, DeFrja and Delbono (1989) show that the presence of a public firm increases welfare when there are only a few Cournot rivals but decreases welfare when there are a larger number of rivals. While allowing the public firm to lead suggests that privatization will reduce welfare, this result need not emerge in a model with foreign firms (Fjell and Heywood, 2004). Within the spatial context, Cremer et al. (1991) examine Hotelling (1929) pricing and location on a line segment. They show that the presence of a public firm actually harms welfare when the total number of firms is more than two and less than six, the type of tight oligopolies governments might be most interested in controlling.

In this paper we modify the model of spatial price discrimination used by Heywood and Ye (2009) to show that the presence of the public firm improves social welfare. In that model the presence of the public firm as either a leader or a follower routinely limits the extent to which private firms can exploit their timing advantage to locate asymmetrically. This emerges because the public firm cares about the total cost of serving customers (the measure of welfare) and so will move into a smaller market segment accepting less profit if it yields more symmetric, and so lower cost, locations. Knowing this, the private leader originally locates in a less asymmetric fashion. In this way the public firm increases welfare in a model of spatial price discrimination.

The current modification follows a strong tradition within the literature on mixed oligopolies by introducing a foreign firm that competes in the domestic market. The introduction of a foreign firm has proven interesting as Fjell and Pal (1996) show that it can have an ambiguous effect on domestic welfare. On the one hand, increased competition can lower prices increasing consumer surplus. On the other hand, the profit earned by foreign firms is repatriated reducing domestic surplus. When moved to the model of spatial price discrimination, the introduction of the foreign firm can sometimes create a similar trade-off between the benefit of increased product variation and cost of repatriated profit. Yet, at other times the introduction of the foreign firm is completely ignored by the public firm in making location choices. In both cases, the

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reprivatization influence typically dominates lowering domestic welfare. Yet, a critical point is that privatization causes the previously public firm to take account of the foreign firm when locating and the result is that global welfare can increase even as privatization lowers domestic welfare. The conclusion is that a domestic government will rarely allow foreign entry into a market with spatial price discrimination and when it does, it will rarely privatize the public firm even though global welfare may increase.

This showing stands as important because spatial price discrimination remains an important economic phenomenon for markets with large transport costs or with differentiated products. Indeed, Greenhut (1981) presents survey evidence showing that among firms for which transport is at least five percent of costs, such discrimination is “nearly ubiquitous” in the United States, Europe and Japan. In addition to the actual delivery of bulky physical products, such location models may apply to many industries with horizontal differentiation. To the extent that firms can identify the location of consumers, the possibility for discrimination exists. Certainly, newspapers locate along a political spectrum from left to right, cereals vary in sweetness and airlines choose times of day for flights. Thissen and Vives (1988) show that spatial price discrimination emerges as a natural consequence of profit maximization and that such pricing will be adopted instead of uniform mill pricing whenever conditions allow. In turn, this has resulted in the application of such models to a variety of policy questions. For example, Rothchild et al. (2000) examine the consequences of horizontal merger and Gupta et al. (1994) examine pricing within a vertical chain and both examinations are set in a context of the spatial price discrimination. This failure does not flow from a lack of relevance as mixed oligopolies in industries from steel to transport costs or extensive elements of horizontal differentiation.

Moreover, they often do so in the presence of foreign firms. Matsushima and Matsumura (2006) describe the recent entry of foreign banks into Japan’s mixed oligopoly. They emphasize that these entering banks tended to engage in “herd” behavior (in terms of markets entered and products offered) that left the behavior of the public banks largely unchanged.

In what follows, we examine three circumstances. In the first, and to build intuition, we consider simultaneous location. We examine a duopoly of a public firm and foreign private firm to help isolate the fashion in which the public firm ignores the foreign firm in adopting its location and next examine a three-firm case that adds a domestic private firm. In the second section, we model a two-stage location choice first with three firms and then with N firms. In the third circumstance, we model three firms in a fully sequential location model (see Gupta, 1992 for three private firms in a domestic market). In both the second and third cases we compare the consequences of allowing each of the three types of firms (public, domestic private and foreign private) to be a location leader. Our fundamental propositions arise from analyzing the contribution made by the foreign firm to both domestic and global welfare and from considering the consequences of privatizing the public firm.

2. Setting up the models

Consumers are uniformly distributed over a unit line segment. Each consumer has inelastic demand for one unit of the good, with

$$\pi_{\text{r}_i} = \text{Profit of a representative interior firm}$$

$$\text{EGF} = \text{the delivered pricing schedule for a representative interior firm}$$

Fig. 1. Spatial price discrimination.

reservation price $r$. We assume $r$ is sufficiently large that it is profitable to serve all customers. There are N firms in the market with one public firm, one foreign firm and $N-2$ domestic private firms. All firms have identical production technologies with the marginal cost of production assumed to be constant and normalized to be zero without loss of generality. The number of firms is fixed in order to focus on issues of market power. Transport cost is $r$ per unit of distance. We adopt the linear market both because it is common and also because it provides a distinct advantage for leadership (compared to a circular market).

The ability to move first and locate in the corner allows the leader to earn greater profit than those firms located in the interior (not the corner) of the market.

Let $L_i$, $i = 1, 2, \ldots, N$, denote the location of firm $i$ on the unit line segment, where $L_{i+1} = L_i + 1$ (see Fig. 1). We will index the type of firms with superscripts for public, domestic private and foreign private ($p$, $d$, $f$). Thus, the cost for firm $i$ to supply all consumers in the line segment $x_0$ to $x_1$ is $C_i = \int_{x_0}^{x_1} c_i dx$ where $c_i = c_i(x - L_i)$. The equilibrium delivered price schedule for any consumer located at $x$ is as follows:

$$p^*(x, L_1, L_{i+1}) = \begin{cases} \frac{t(L_2 - x)}{L_1} & \text{if } x \leq L_1 \\ \max\{\frac{t(x - L_1)}{L_{i+1}}, \frac{t(L_{i+1} - x)}{L_{i+1}}\} & \text{if } L_1 \leq x \leq L_{i+1} \\ \frac{t(L_i - x)}{L_N} & \text{if } x \geq L_N \end{cases}$$

(1)

As shown in Fig. 1, the price schedule is the lower envelope of rival’s cost curves as shown by EGF. We follow convention by assuming that customers who are indifferent (they face the same delivered price from more than one firm) purchase from the closest firm.

Consequently, the profit of an interior firm $i$ is:

$$\pi_i(L_{i-1}, L_i, L_{i+1}) = \int_{L_{i-1}}^{L_{i+1}} (c_i - c) dx + \int_{L_{i-1}}^{L_i} \frac{t}{L_1} (c_i - c) dx$$

$$= \frac{t}{4} (L_{i+1} - L_i)^2 - \frac{t}{4} (L_i - L_{i-1})^2$$

(2)

The profits of corner firms 1 and $N$ are:

$$\pi_1(L_1, L_2) = \int_0^{L_2} (c_2 - c_1) dx$$

$$= \frac{t}{2} L_2^2 - \frac{t}{2} L_1^2 - \frac{t}{4} (L_2 - L_1)^2$$

$$\pi_N(L_{N-1}, L_N) = \int_{L_{N-1}}^{L_N} (c_N - c) dx$$

$$= \frac{t}{2} (1 - L_{N-1})^2 - \frac{t}{2} (1 - L_N)^2 - \frac{t}{4} (L_N - L_{N-1})^2$$

(3)
The total transport cost remains independent of firm type and is:

\[ TC = \sum_{i=1}^{N} C_i = \frac{L_2^2}{2} + \frac{L_1^2}{2} (1-L_0)^2 + \sum_{i=1}^{N-2} \frac{L_i^2 (L_{i+1} - L_i)^2}{4}. \tag{4} \]

The global welfare function becomes:

\[ W(L_1, L_2, \ldots, L_N) = r - TC \tag{5} \]

The essence of the public firm’s problem is to maximize domestic welfare, the sum of consumer surplus and domestic profit. We express this as global welfare minus the profit of the foreign firm.

\[ D(L_1, L_2, \ldots, L_N) = W(L_1, L_2, \ldots, L_N) - \pi_f^t \tag{6} \]

In each of the next two sections, we compare location models for a mixed oligopoly with a foreign firm to those for a mixed oligopoly without a foreign firm and to those for a private oligopoly that has no public firm. Before moving to those comparisons, we remind readers of the solution to the simultaneous location model with only private firms and use this solution to present the analogous solution for a mixed oligopoly model with simultaneous location.

**Lemma 1.** A private oligopoly engaging in spatial price discrimination and locating simultaneously generates symmetric locations.

Given \( L_i \) and \( L_i+1 \) from Eq. (2), the optimal location for any interior firm \( i \) is \( L_i = \frac{L_2 - L_1}{2} \) yielding \( L_i = L_i - 1 = L_i - l_i \). Similarly, from Eq. (3) the optimal locations of the corner firms will be \( L_1 = \frac{L_2}{2} \) and \( L_N = \frac{L_2}{2} + \frac{X_1}{C_0} \). Simultaneous solution yields that all firms locate symmetrically. This is the well-known result that spatial price discrimination results in symmetric locations and lowest total cost, highest welfare. See Lederer and Hurter (1986) for the original two-firm proof.

3. **Simultaneous entry**

This section begins by examining a simultaneous location duopoly with a public firm and foreign private firm. It then considers a simultaneous location model with three firms, a public firm, foreign private firm and domestic private firm. It concludes with a robustness check of the three-firm case that introduces elastic demand. This check reinforces the insight that the reaction function of public firm does not include the location of the foreign firm.

3.1. Duopoly with a foreign firm

In this subsection we consider the case of a duopoly consisting of only the public firm and the foreign firm. This consideration quickly conveys the logic that the public firm will ignore the presence of the foreign firm in making location choices. In stage one, firms enter and choose a location in the market. In the second stage, firms simultaneously announce the delivered price schedule. The public firm maximizes domestic welfare (Eq. (6)) and the foreign firm maximizes its own profit. As it is immaterial, we assume the foreign private firm locates to the left.

**Proposition 1.** The addition of a foreign firm has no influence on domestic welfare. The location of the public firm and the value of domestic welfare in a duopoly with a foreign firm are identical to those emerging from a domestic monopoly.

**Proof.** The public firm’s welfare maximization is equivalent to cost minimizing its own cost of serving the market. This yields a dominant strategy \( L_1^* = \frac{L_2}{2} \). The best response function of the foreign firm is \( L_1^* = \frac{L_2}{2} \), which implies that \( L_1^* = \frac{L_2}{2} \).

We note that given any location for the public firm, domestic welfare is completely invariant to the location of the foreign private firm. While the location of the public firm generates the cost curve that serves to limit the price of the foreign firm, all profit from the foreign firm is repatriated and so its location does not influence domestic welfare. As shown in Fig. 2, the presence of the foreign firm merely transfers what would have been part of the cost of serving customers into foreign profit leaving welfare unchanged (the shading shows this transfer). Thus, the public firm merely locates to minimize cost and maximize domestic welfare as if the foreign firm did not exist.

In the traditional non-spatial mixed duopoly, the presence of a foreign firm increases domestic welfare as the increase in consumer surplus exceeds the expatriated profits (Fjell and Pal, 1996). A critical insight in this simple case of spatial price discrimination is the presence of the foreign firm does nothing to change domestic welfare.\(^5\)

**Proposition 2.** In the spatial mixed duopoly with a foreign firm, the privatization of the public firm reduces domestic welfare but increases global welfare.

**Proof.** Privatization results in symmetric locations, \( L_1^* = \frac{L_2}{6} \) and \( L_2^* = \frac{L_2}{4} \), and welfare follows accordingly:

\[ D \left( L_1^* = \frac{1}{6}, L_2^* = \frac{1}{2} \right) > D \left( L_1^* = \frac{1}{4}, L_2^* = \frac{3}{4} \right) \]

\[ W \left( L_1^* = \frac{1}{4}, L_2^* = \frac{3}{4} \right) > W \left( L_1^* = \frac{1}{6}, L_2^* = \frac{1}{2} \right). \]

Thus, the equilibrium with a public firm has higher domestic welfare but lower global welfare when compared to that emerging with two privatized firms.

3.2. Oligopoly with both a domestic and a foreign private firms

In an international mixed oligopoly with both a domestic private firm and a foreign private firm, the public firm no longer has a dominant location strategy but it retains a best response function that depends only on the location of the domestic private firm. The location of the foreign private firm remains irrelevant. For example if the public firm is to the right of the domestic firm, the best response is \( L_2^* = \frac{L_2}{6} + \frac{L_2}{4} \) even if the foreign firm is in the middle.

We denote the equilibrium \( \mathcal{E} = \{D, F, L\} \) as the locations of the public, domestic private and foreign private firms respectively. In simultaneous entry, there exit three equilibria: \( \mathcal{E}_1 = \left( \frac{L_2}{6}, \frac{L_2}{4}, \frac{L_2}{3} \right) \), \( \mathcal{E}_2 = \left( \frac{L_2}{6}, \frac{L_2}{4}, \frac{L_2}{3} \right) \) and \( \mathcal{E}_3 = \left( \frac{L_2}{6}, \frac{L_2}{4}, \frac{L_2}{3} \right) \). The associated domestic welfare becomes \( D_1 = r - \frac{L_2}{6} D_2 = r - \frac{L_2}{4} \) and \( D_3 = r - \frac{L_2}{3} \). The associated global welfare becomes \( W_1 = r - \frac{L_2}{6} W_2 = r - \frac{L_2}{4} \) and \( W_3 = r - \frac{L_2}{3} \). The different equilibria reflect three relative positions of the public firm: 1) the public firm locates at the right corner next to a domestic private firm, 2) the public firm locates at the right corner but next to the foreign, and 3) the public firm locates at the middle. The variation\(^5\)
results because the public firm’s location decision differs depending
on the type of firm that is its direct neighbor.

**Proposition 3.** In a mixed oligopoly locating simultaneously, the
addition of the foreign firm can lower domestic welfare but will never
increase domestic welfare.

**Proof.** Equilibrium in a mixed duopoly with a domestic private
firm is \( L_1^F = \frac{1}{2}, L_2^F = \frac{1}{2} \). The welfare results follow immediately:
\[ D(L_1^F = \frac{1}{4}, L_2^F = \frac{1}{4}) = D_3 > D_2 > D_1. \]

Note the interesting point that the three-firm equilibrium that
maximizes domestic welfare yields exactly the domestic welfare that
emerges from just the public firm and the domestic firm locating
simultaneously. This matches the duopoly result where the best the
public firm could do to maximize domestic welfare is to minimize its
own transport cost for serving the entire market. Now the best that
the public firm can do is to locate together with the domestic firm so as to
minimize the transport cost of two firms serving the entire market.

**Proposition 4.** In a mixed oligopoly with a foreign firm locating
simultaneously, privatization never increases domestic welfare and never
decreases global welfare.

**Proof.** From the Lemma, the private equilibrium is \( \rho_{prv} = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3}) \)
which is identical to \( \rho_1 \). The welfare results follow immediately:
\[ W_{prv} = W_1 > W_2 > W_3 \] and \[ D_{prv} = D_1 < D_2 < D_3. \]

We note that the pattern presented can be easily generalized to any
number of domestic and foreign private firms. Whenever the public
firm locates next to only domestic firms, the public firm locates
symmetrically in an identical fashion to any other private firm
(Heywood and Ye, 2009). Whenever the public firm locates next to a
foreign firm, the location of that foreign firm does not enter its best
location response function. As a consequence, asymmetric locations
emerge that increase domestic welfare but at the expense of global
welfare. Privatization necessarily can never increase domestic welfare
but never decrease global welfare.

3.3. Discussion and robustness check

It is worth emphasizing that as long as the entry of the foreign firm
causes the other firms in the market to move, two offsetting influences
will be observed. First, the additional firm adds to the product
variation increasing consumer welfare. Second, the profit of the

foreign entrant is expatriated lowering domestic welfare. These two
influences are shown in Fig. 3 that illustrates the foreign firm on the
left and the public firm on the right, \( \rho_1 = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3}) \). The increased
product variation generates regions A that increase domestic welfare
turning what was cost into domestic profits. This is partially offset by
the smaller losses in domestic welfare turning what was profit in to
costs, the regions B. Finally, this is further offset by the expatriated
profit but note that some of that expatriated profit comes from what
was previously profit, C1, but some has no welfare effect as it comes
from what was previously cost, C2. The point again, is to emphasize
that when the private firms move in response to the presence of the
foreign firm there are offsetting influences on welfare. These
influences are analogous to those in the non-spatial context in
which foreign entry lowers the price increasing consumer surplus but
expatriates profit lowering domestic welfare (Fjell and Pal, 1996). It is
these offsetting influences that make strategic trade issues in a mixed
oligopoly interesting (Pal and White, 1998). We retain the similar
offsetting influences of product variation and expatriates profit in all
cases of spatial price discrimination in which the public firm is not
located next to a foreign firm in the corner.

When the public firm is located next to the foreign firm in the
corner, none of the domestic firm locations is influenced by the
presence of the foreign firm and this trade-off is lost. Thus, in the
duopoly or in \( \rho_3 = (\frac{2}{3}, \frac{1}{3}, \frac{1}{3}) \), the domestic firms remain in their
original locations and the presence of the foreign firm serves only to
expatriate profit. We recognize that this may be considered an
extreme view but it appears to be a genuine function of assuming
spatial price discrimination on a linear market. Specifically, as we
describe, it is robust to allowing elastic demand.

Hamilton et al. (1989) model spatial price discrimination in which
at each location on a line segment there exist linear demand schedules
rather than inelastic demand for a single unit. They consider a duopoly
in which each competitor first locates and then sets a location-specific
price schedule given the price schedule of its rival. The equilibrium
locations depend on the transport cost and are shown to be
symmetrically inside the first and third quartiles. We examine the
duopoly case and the specific three-firm case with the public firm in
the middle and the foreign firm on the left using this model with

![Fig. 2. Equilibrium of mixed duopoly with a foreign firm.](image-url)
elastic demand. These cases correspond to the circumstances with inelastic demand in which the presence of the foreign firm merely results in expatriated profit. As Appendix A shows, the locations of the firms typically depend on the transport cost and, as in the fully private case of Hamilton et al. (1989), the firms locate closer together in the face of elastic demand. Critically, the locations and price schedules of the domestic firms remain unchanged by the presence of the foreign firm. The foreign firm simply expatriates as profit what used to be costs. This does not imply that the presence of elastic demand leaves locations unchanged rather it confirms that when the foreign firm locates between the public firm and the corner, the domestic firms continue to ignore its presence even with elastic demand.

4. Location leadership: Two-stage entry

The idea of sequential entry has a long history in location models as firms are rarely seen to enter simultaneously (Hay, 1976, Prescott and Visscher, 1977). In this section, we separately examine each type of firm acting as a location leader with the other two firms locating as simultaneous followers. In the next section we consider a fully sequential three stage location model. In modeling a two-stage location game, Heywood and Ye (2009) show that if the public firm leads, first best cost minimizing locations result. If a domestic private firm locates first, the presence of the public firm acts to constrain the resulting location asymmetries as the public firm’s critical “no-jump” condition is more binding on the first mover than are the no-jump conditions of the private firms. We now return to this with the presence of a foreign private firm.

A single subscript identifies the type of firm leading in each of the three cases. We assume the leader locates at the left corner to capture extra corner profit if it can. The maximization and the relevant no-jump conditions generate the subgame perfect Nash equilibrium for each case. As seen in the previous section, a critical issue is whether the location of the foreign firm enters the reaction functions of the other firms. It never directly enters the reaction function of the public firm and enters indirectly only when the location of the domestic private firm is influenced by the location of the foreign firm.

In what follows, we initially develop the three-firm case by finding the potential equilibria associated with each firm leading (public, domestic private and foreign private). We then consider which of the potential equilibria might emerge endogenously. We conclude by considering the N-firm case with multiple private domestic firms.

4.1. A public leader

We identify this case as LFF, meaning that the public firm is a leader, the private domestic firm (listed second) is a follower and the private foreign firm (listed third) is also a follower. In order to minimize the domestic welfare loss from the foreign firm, the public firm locates in the corner and moves toward the center squeezing the foreign firm’s profit. However, the resulting asymmetric locations increase total transport costs lowering social welfare. Ultimately, the public firm locates so as to balance these two forces. We consider the two cases in which the foreign firm is the interior firm and in which the foreign firm is the corner firm. The no-jump condition from the corner firm is dominated by the no-jump condition from the interior firm for both cases and thus the no-jump condition equates the interior profit of a private firm, \( \pi_i = \frac{p_i - c_i}{2} \), with its profit if it jumps to the left corner, \( \pi_i = \frac{p^0 - c_i}{2} \). This implies that \( p^0 = 0.329 \). The unconstrained welfare maximization, \( L^0 = 0.265 \), is such that \( L^0 < L^1 \) for a foreign interior location. Moreover, the unconstrained welfare maximization, \( L^0 = 0.306 \), is such that \( L^1 > L^0 \) for a foreign interior location. Thus, the first order condition of the welfare maximization, rather than the no-jump condition, is binding in both cases and \( L^0 = L^1 \) becomes the leader’s equilibrium location for the foreign firm in the interior and \( L^1 = L^0 \) for the foreign firm in the right corner. This generates

\[
D_l^p = r - 0.1324t \quad D_l^R = r - 0.1528t \\
W_l^p = r - 0.0891t \quad W_l^R = r - 0.0949t
\]

where superscripts \( l \) and \( R \) indicate the foreign firm in the interior location and the right corner location respectively and subscripts \( p \) indicate the public leader. See locations and other equilibrium values in the LFF entry of Table 1.

4.2. A domestic private leader

As before the leader locates in the left corner if it can and we identify the case of the domestic private leader as FFL. Given the private leader’s location, the public firm will choose the welfare-maximizing location. This location remains independent of the foreign firm’s location such that the best response function of the public firm is \( L_p = \frac{1}{3} + \frac{r}{t} \) given \( L^0 \). Knowing this, the domestic private leader moves as far as possible towards the center without allowing a jump of the foreign firm (note that for the foreign firm, its interior location dominates its right corner location). The interior foreign firms’ profit without jumping is \( \pi_l = \left( \frac{L^0 - L^1}{2} \right) \frac{L^1}{18} \) given \( L^0 \) above and this is equated to the profit it makes if it jumps, \( \pi_l = \frac{L^1}{2} \). The no-jump constraint is binding and implies \( L^0 = 0.290 \), and yields:

\[
D_d = r - 0.1261t \quad W_d = r - 0.0980t
\]

See the locations and other equilibrium values in FFL entry in Table 1.

4.3. A foreign private leader

The foreign leader attempts to locate in the left corner of this case identified as FFL. This gives rise to two equilibria depending on the relative locations of the other firms. First, when the public firm is left of the domestic private firm, the foreign firm must locate with the realization that the public firm ignores its location. Knowing this, the foreign firm actually gives up the corner location and adopts the center position. It weights the profit of an extreme left position and the public firm locating at \( L_p = \frac{t}{2} \) with a central location that pushes the domestic private firm right. The latter dominates. Thus, the domestic private firm’s best response becomes \( L^1 = \frac{1}{3} + \frac{r}{t} \) and the foreign firm’s profit maximization generate its optimal location, \( L^1 = \frac{L^0 + L^1}{2} \). These three equations solved simultaneously generate the first equilibrium.

Second, when the public firm is on the right of the domestic private firm, the profit-maximizing foreign firm would choose its optimal location on the left such that neither the public or domestic private

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Equilibria of the eight timing cases.</th>
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<tbody>
<tr>
<td>( \varepsilon = {p^L, L^1, L^2} )</td>
<td>( n^p )</td>
</tr>
<tr>
<td>LLL/FFF 1</td>
<td>(5/6, 1/2, 1/6)</td>
</tr>
<tr>
<td>2</td>
<td>(5/7, 1/3, 1/7)</td>
</tr>
<tr>
<td>3</td>
<td>(1/4, 3/4, 1/12)</td>
</tr>
<tr>
<td>LIF</td>
<td>(0.306, 0.583, 0.861)</td>
</tr>
<tr>
<td>1</td>
<td>(0.283, 0.833, 0.5)</td>
</tr>
<tr>
<td>2</td>
<td>(1, 0.833, 0.5)</td>
</tr>
<tr>
<td>3</td>
<td>(0.833, 0.5, 0.167)</td>
</tr>
<tr>
<td>LFL</td>
<td>(1/2, 1/3, 1/6)</td>
</tr>
<tr>
<td>1</td>
<td>(0.275, 0.716, 0.484)</td>
</tr>
<tr>
<td>2</td>
<td>(0.763, 0.290, 0.527)</td>
</tr>
</tbody>
</table>

The numbers 1, 2 and 3 denote equilibria associated with three relative positions of the public firm as in Section 3.2. Similarly I and L and R denote equilibrium associated with location of the foreign firm as interior, left or right as in Sections 4.1 and 4.3.
4.4. Comparing the results

We start considering the implications of the different leadership cases by noting that domestic welfare is always lower when the foreign firm is able to retain a corner position. This circumstance typically results in greater expatriation of profit and less symmetry between the two domestic firms. Nonetheless, the anticipated entry of the foreign firm can limit the asymmetry caused by a private domestic leader.

Proposition 5. In a three-firm mixed oligopoly with a location leader, the addition of a foreign firm decreases domestic social welfare when the public firm leads, but increases domestic social welfare when the domestic private firm leads.

Proof. Without a foreign firm, the welfares of a domestic mixed duopoly with a public and a private leader are: \( D_{fl} = r - 0.125t \) and \( D_{fl} = r - 0.14t \). It can be easily verified \( D_{fl} - D_{fl} > D_{fl} - D_{fl} > D_{fl} - D_{fl} > D_{fl} - D_{fl} \).

Thus, if the public firm locates first, a government interested in domestic welfare maximization opposes foreign entry as expatriation clearly causes that welfare to decline. When a private domestic firm locates first, the results are less clear-cut a priori. The presence of the foreign entrant limits the extent to which the domestic private leader can asymmetrically offsetting the loss through expatriation. In weighing these, the reduction in asymmetry dominates.

Proposition 6. In a three-firm international mixed oligopoly with a location leader, privatization always lowers both domestic welfare and global welfare.

Proof. The domestic welfares after privatization with a domestic leader associated with interior and corner foreign location and a foreign leader are respectively: \( D_{fl}^{prv} = r - 0.1351t \), \( D_{fl}^{prv} = r - 0.1532t \) and \( D_{fl}^{prv} = r - 0.2054t \). The global welfare after privatization is \( W_{fl}^{prv} = r - 0.167t \). Therefore, we have \( D_{fl}^{prv} - D_{fl}^{prv} > 0, D_{fl}^{prv} - D_{fl}^{prv} > 0, D_{fl}^{prv} - D_{fl}^{prv} > 0, D_{fl}^{prv} - D_{fl}^{prv} > 0, D_{fl}^{prv} - D_{fl}^{prv} > 0, W_{fl}^{prv} - W_{fl}^{prv} > 0, W_{fl}^{prv} - W_{fl}^{prv} > 0, W_{fl}^{prv} - W_{fl}^{prv} > 0 \) and \( W_{fl}^{prv} - W_{fl}^{prv} > 0 \).

Privatization decreases welfare regardless of the leadership structure. Again, a government interested in domestic welfare maximization is unlikely to allow foreign entry and when it does, it is unlikely to privatize. Such privatization causes both domestic and global welfare to decline.

These results can be compared to the privatization results from the otherwise similar model that doesn’t include a foreign firm (Heywood and Ye, 2009). In that case, privatization lowered welfare because the public firm acted to constrain the location advantage of a private first mover. This happens because the public firm cares about symmetry rather than just its own profit. With the presence of the foreign firm, the public firm still acts to constrain a private leader but it also locates asymmetrically itself by only being concerned about the presence of the domestic firms. Privatization causes both of these influences to vanish.

4.5. Endogenous leadership: The case of three firms

In this subsection we explore the issue of endogenous timing. The issue is what timing might emerge from the two-stage game we have modeled above. We modify slightly the observable delay game of Hamilton and Slutsky (1990) in which firms first announce the stage that they will choose their quantities in and are then committed to this choice before they actually produce. The issue of endogenous timing has been of interest to researchers examining mixed oligopolies starting with Pal (1998). While foreign firms have been examined in this context (Matsumura, 2003; Lu, 2006a,b), no one has examined endogenous timing in our context of spatial price discrimination. To do so, we imagine that the observable delay requires firms to identify the stage of their entry and they are committed to this stage before they actually adopt a location.

Using the solutions from the previous sections, we can identify the locations under the possible timing choices. Note that three firms and two periods result in eight possible timing choices (2^3). When all three firms commit to being a leader (LLL) or all three firms commit to being a follower (FFF), the quantities of all three firms are adopted simultaneously and the equilibria become those in Section 3.2. When the domestic private firm leads, there is only a single equilibrium as shown in Section 4.3. Yet, when either the public or foreign firm leads, two potential equilibria emerge depending on whether or not the public and foreign firms are positioned next to each other.

To these solutions we add the three timings representing the cases when two of the firms lead and one follows. In two of these additional timings, there is only a single equilibrium with the foreign firm next to the public firm. In the final case, there are two potential equilibria depending upon whether or not the public firm is next to the foreign firm. We list the equilibria of the eight timings in Table 1 and demonstrations are available from the authors. The timing is critical with the measure of domestic welfare varying from a low of \( r - 0.1667t \) to a high of \( r - 0.125t \). The variation in the pattern of location and welfare comes from the distinction in both positioning and timing itself.

We consider each critical positioning in turn. First, under the assumption that the public firm is not positioned next to the foreign firm, we examine all eight possible timings. Second, under the assumption that the public firm is positioned next to the foreign firm we examine all eight possible timings. Viewed in this way, in which the positioning is taken for granted, the only timings that are routinely excluded are those of simultaneous entry.

Proposition 7. i) When the public firm is not next to the foreign firm, the endogenous timing equilibria include FFL, LFL and FFL. ii) When the public firm is located next to the foreign firm and simultaneous location assumes the foreign firm at the corner, the endogenous timing equilibria include LLF and FFL. When the public firm is located next to the foreign firm and the simultaneous location assumes the foreign firm in the middle, the endogenous timing equilibrium is FLL.

Proof. See Appendix B.

While these endogenous timings depend upon the relative positioning of the firm types, several things are evident. First, as indicated, the simultaneous location timing is never adopted. Second, of the three cases of a single leader examined in detail in the earlier subsections, only a foreign leader endogenously emerges. Moreover, five of the six possible equilibria involve the foreign firm as a leader. Third, all of the cases involving two leaders emerge at least once across the various relative positions. These generalizations fit some of the characteristics of the endogenous timing results from outside the spatial context. For example, Lu (2006b) shows that the public firm is

\[ \text{6 Indeed as of December 1, 2008, Pal (1998) had more than fifty cites in Google Scholar.} \]
never a unique leader when there are both domestic and foreign private firms, a result mimicked in our spatial context. Note that the failure of simultaneous locations to emerge endogenously removes both the highest and lowest domestic welfare entries from Table 1. Nonetheless, the degree of variation across the endogenous timings remains substantial as indicated by Table 1.

4.6. The N-firm case

Our original model allows for N firms of which \( m = N - 2 \) are private domestic firms. We explore this case by continuing to imagine a two-stage location game in which any of the three types of firms may be the leader. While there are many different potential positions in such a general model, we emphasize that the three-firm results largely carry over. We demonstrate in this subsection the most critical difference which is the ability of privatization to increase global welfare even as it reduces domestic welfare. While it remains the case that privatization always decreases domestic welfare, it can increase global welfare when there is a foreign private leader and a relatively large number of domestic private firms.

As in the case of three firms, this generates three timings and in the case of the public or foreign firm leading two sets of equilibria depending on whether or not the public firm is next to the foreign firm. In all of these timings except that of the foreign firm leading, the critical privatization results from the case of three firms continue to hold. In the case of a foreign leader, this result must be modified when the foreign firm locates next to the public firm. Although privatization continues to lower domestic welfare, it can now increase global welfare.

The foreign leader would like to enter on the left but recognizes that it will be next to the public firm. As before, the public firm ignores the position of the foreign firm and its reaction function (which depends on the adjacent domestic firm) causing it to adopt the left corner position. Knowing this, the foreign leader adopts the second position but pushes the domestic private firms as far to the right as possible. The equilibrium is presented in Appendix C.

**Proposition 8.** Privatization increases global welfare for an interior foreign private leader if \( m \geq 7 \); otherwise, privatization decreases global welfare.

**Proof.** The global welfare for an interior foreign private leader is

\[
W_f^I = \begin{cases} 
\frac{2m + 21 - 4\sqrt{6}}{2(2m + 6 - 1)} & \text{if } m > 2 \\
\frac{t(2m + 3)(m + 1)}{12(2m + 1)^2} & \text{if } m \leq 2 
\end{cases}
\]

where the first segment follows because the non-jump condition is binding and the second follows because it no longer binds. Global welfare after privatization is

\[
W_f^{nv} = r - \frac{2m + g(m)}{2(2m + 3 - \sqrt{6})}. \]

Solve \( W_f^I - W_f^{nv} = 0 \) for \( m \). The only positive root is \( m = 6.37 \). Checking integer values of \( m \) above and below the critical value confirms that \( W_f^I - W_f^{nv} < 0 \) as \( m \geq 7 \) and \( W_f^I - W_f^{nv} > 0 \) as \( m < 7 \). □

Privatization decreases domestic welfare regardless of the market structure. Yet, privatization can increase global welfare when the number of private domestic firms is large. The equilibria with the public firm have the \( m \) private firms pushed further to the right than they would be in the fully private equilibrium. At the same time, the two firms on the left, the foreign and public firm, locate more symmetrically than do the foreign and newly privatized firm in the post-privatization equilibrium. When \( m \) grows, the fact that two firms on the left locate more symmetrically becomes less important than the fact that the \( m \) private firms are pushed further to the left. In this case, a government interested in domestic welfare maximization is unlikely to allow foreign entry and when it does, it is unlikely to privatize. Such privatization causes domestic welfare to decline despite the fact that it causes global welfare to increase.

5. Fully sequential entry

As seen, two-stage entry but with three of more firms gives rise to the importance in some cases of the relative positioning of the different types of firms (whether or not the public firm and foreign firm are next to each other). Fully sequential entry does not give rise to this issue as only a single firm locates at each stage. Gupta (1992) presents the case of a private firm oligopoly of three firms in a model of spatial price discrimination. We recognize that efforts to generalize sequential location beyond three firms have proven difficult (Gupta, 1992) and limit our attention of fully sequential locations to the case of three firms. Using the critical “no-jump” constraints, she presents the equilibrium: \( L_1 = 0.275, L_2 = 0.5, L_3 = 0.725 \). Heywood and Ye (2009) modify this model to include a public firm that maximizes welfare. This modification results in the same locations as Gupta if the public firm locates last but results in more symmetric locations and greater welfare if the public firm locates either first or second. We now modify the model further to investigate a fully sequential three-firm game that includes both a public firm and a foreign private firm.

We continue to denote the three-firm equilibrium as before, but now add subscripts to identify the order of location. As an example, \( \epsilon^{pf} = (L', L'`, L') \) indicates that the domestic firm moves first, the public firm moves second, and the foreign firm moves last. The arguments in the parentheses remain the locations of the public, domestic private and foreign private firms respectively. Deriving equilibrium for each case requires being cognizant of the no-jump conditions. We begin with the case in which the public firm leads and then move to the other two firms as leaders. We wait till all cases have been solved to draw conclusions regarding welfare and privatization.

5.1. The public firm locates first

There exist two cases: either the domestic private firm or the foreign private firm locates second. Starting with the domestic private firm locating second, the equilibrium locations are solved by backward induction. Both domestic firms will locate so as to force the third firm to locate at the middle \( L' = \frac{L'` + 2L'}{2} \). This maximizes the profit of the private domestic firm and minimizes the loss of domestic surplus. Assuming the first mover locates at the left, the no-jump constraint equates the profit of the foreign firm locating in the middle \( n = \frac{L'}{2} \) to that of it locating to the right of the domestic private firm \( n = \frac{1-L'}{2} \). Solving this equality gives the reaction function of the domestic private firm that forces the foreign firm to the middle. Substituting that reaction function into that for the foreign firm yields the best response functions of the two private firms to the public firm: \( L' = \frac{8 - 2L'}{2} + \frac{2L'}{2} - 1 \) and \( L'` = \frac{4 - L'`}{2} + \frac{1-L'}{2} \). These are returned to the domestic welfare function and maximization generates the equilibrium location of public firm and, through substitution, of the private firms: \( \epsilon^{pf} = (0.252, 0.716, 0.484) \).

When the foreign firm locates second, it cannot be pushed to the middle. As a consequence, the public firm maximizes domestic welfare by locating exactly in the middle forcing the other two firms to either side: \( \epsilon^{pf} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \). If it fails to locate exactly in the middle, the foreign firm would use its leadership to force the domestic private firm into the middle position reducing domestic welfare. The resulting welfares of the two cases of public firm leadership follow from the equilibrium locations:

\[
\begin{align*}
D_{pdf} &= r - 0.1259r, \quad W_{pdf} = r - 0.0990r \\
D_{pdf} &= r - 0.1667r, \quad W_{pdf} = r - 0.0833r
\end{align*}
\]
5.2. The domestic private firm locates first

The domestic private firm locates first in the left corner with the remaining subgame between only the public and foreign firms. As a consequence, the public firm’s location does not depend on whether it locates second or third and remains independent of the foreign firm’s location: \( L^o = \frac{1}{2} + \frac{r}{r^*} \). Knowing this, if the foreign firm locates to the right of the private domestic leader, it is forced to the middle regardless of whether it moves before or after the public firm: \( L^f = \frac{1}{2} + \frac{r}{r^*} \). The only no-jump condition merely guarantees that the foreign firm does not locate to the left of the domestic leader. This equates the profit earned by the foreign firm in the middle, \( \pi^m = \frac{(1-\mu)^3}{2} \), with that earned on the left of the domestic private firm

\[
D_{dpf} = D_{dfp} = r - 0.1261t
\]

5.3. The foreign firm locates first

The foreign firm locates first in the interval \([0, 0.5]\). The public firm again ignores the foreign firm’s location. If the public firm locates last, it does so to minimize the cost of serving the market left of the domestic private firm: \( L^o = \frac{r}{r^*} \). This behavior necessarily pushes the public firm to the left of the foreign firm generating the reaction function of the domestic private firm: \( L^p = \frac{1}{2} + \frac{r}{r^*} \). Substituting these reaction functions into the profit function of the foreign firm shows that the best the foreign firm can do is locate in the exact middle to give itself as large an interior market as possible. The resulting locations do not change if the public firm moves second rather than third because the foreign firm remains between the two domestic firms (a demonstration is available upon request). This generates the equilibrium, \( \mu_{dpd} = \mu_{pdf} = (0.278, 0.833, 0.5) \), and the associated welfare:

\[
D_{pdf} = D_{dfp} = r - 0.130t
\]

Thus, the influence of adding a foreign firm on domestic welfare can be positive for the first time. While it remains negative when the public firm can locate first, this reverses when the domestic private firm leads. Even though the public firm ignores the location of the foreign firm, the private domestic firm locates more nearly symmetrically because of the presence of the foreign firm. This causes domestic welfare to increase. Thus, despite the presence of a domestic welfare-maximizing public firm, a governmental authority with the same objective would prefer foreign entry but only when the public firm is not the location leader.

We now take the three-firm equilibria for granted and imagine the consequences of privatizing the public firm. The locations of the privatized three-firm case become those of Gupta but domestic welfare depends on the location of the foreign firm. The domestic welfares with foreign firm locating at corner or middle and the global welfare are

\[
D_{pdf} = r - 0.1755t, D_{dfp} = r - 0.1263t \text{ and } W_{pfr} = r - 0.0884t.
\]

Thus, as before, privatization can never increase domestic welfare. Yet, the fully sequential model with three firms indicates that in at least one timing, privatization will lower not only domestic welfare but also global welfare. When the public firm moves first followed by the foreign firm, domestic welfare maximization requires putting the foreign firm in as small an area as possible and it cannot be forced between the two domestic firms. As a consequence, the public firm locates in the middle maximizing not only domestic but also global welfare. Privatization of all firms results in a lower global welfare in this case. In all other cases, privatization improves global welfare.

While privatization does generate one case in which global welfare falls, the trade-off between global welfare and domestic welfare emerges very clearly. If one simply ranks all six timing cases prior to privatization, this is brought into sharp relief. The ordering for domestic welfare is exactly the reverse of that for global welfare:

\[
D_{pdf} < D_{dfp} < D_{pdf} = D_{dfp} < D_{pdf}
\]

5.4. Comparisons and discussion

In five of the six timings, the relative position of the firms in the market remains identical.

Proposition 9. In a three-firm mixed oligopoly with fully sequential location choices, the foreign firm is always pushed to the middle except when it follows a public leader.

Proof. As derived above the foreign firm is in the middle with the exception of the second timing: \( \mu_{pdf} = (0.252, 0.716, 0.484) \), \( \mu_{pdf} = (0.278, 0.833, 0.5) \), \( \mu_{dfp} = (0.290, 0.763, 0.527) \) and \( \mu_{fdp} = (0.278, 0.833, 0.5) \).

The result of the foreign firm is often pushed to the middle emerges because the location choice of the public firm is unaffected by the location of the foreign firm. This removes the advantage of being an earlier mover from the foreign firm so that even when it leads, the public firm locates so as to eliminate the advantage of the foreign firm being in the corner.

We now turn to examining the role of the foreign firm and influence of privatization on welfare. In starting we recall that in a duopoly between a domestic private firm and a public firm, the order of location matters. When the public firm locates first it can establish first best locations of \( \frac{1}{2}, \frac{1}{2} \) with welfare \( D_{pdf} = r - 0.125t \). When the public firm locates second, the Gupta (1992) equilibrium emerges \( (\frac{1}{2}, \frac{1}{2}) \) with welfare \( D_{pdf} = r - 0.14t \). With these as starting points, we identify the influence of adding a foreign private firm.

Proposition 10. In a three-firm mixed oligopoly with fully sequential location choices, the addition of a foreign firm decreases domestic social welfare when the public firm locates first, but increases domestic social welfare when the domestic private firm locates first.

Proof. The first claim follows from Eq. (10): \( D_{pdf} > D_{dpf} > D_{pdf} \). The second claim follows from Eq. (11): \( D_{dpf} < D_{pdf} < D_{pdf} \).
regulatory effort to improve domestic welfare by altering timing will diminish global welfare.

6. Conclusions

Earlier work with a model of spatial price discrimination has yielded unambiguous results regarding the role of a public firm in a domestic mixed oligopoly (Heywood and Ye, 2009). The presence of the public firm routinely increases welfare as the public firm follower will jump into a small corner than a private firm follower given a first stage location leader. Thus, the public firm acts to restrict the location advantage of private first movers. Privatizing the public firm reduces welfare by generating more asymmetric locations.

Allowing entry of a foreign firm into this model requires modifying these results. The entry of the foreign firm lowers domestic welfare. The public firm ignores the presence of the foreign firm in its reaction function and this generates asymmetric locations (although there can be indirect effects on the public firm location as the other domestic firms respond to the entry). As a result privatization will reduce welfare. This suggests that a domestic government may resist entry of the foreign firm and if such entry happens, it will restrict privatization. The fact that privatization can increase global welfare differs from the fully domestic market in which privatization can never increase welfare. Nonetheless, we were able to isolate some cases in which even with the foreign firm, privatization decreases global welfare.

In comparing the results of this model to those outside the spatial context, it is important to emphasize that outside the spatial context the entry of a foreign firm into this model requires modifying these results. Therefore, the welfare function is identical to Eq. (A1) and is independent of the foreign firm’s location. The foreign firm maximizes its profit

\[ \pi_F^* = \int_0^{\epsilon_1} (\epsilon_F - c_1) t (L^F - x) dx \]

Thus, the public firm location remains unchanged while that of the foreign private firm is

\[ L^F_1 = \frac{7t - 12r + 2\sqrt{7t^2 - 28tr + 36r^2}}{14t} \]  

The intuition is that entry of a foreign firm reduces transport cost and cost reduction is the foreign firm’s profit. This foreign profit is shifted out of the country but leaves domestic welfare unchanged. As an illustration, if \( r = 1 \) and \( t = 1/4 \), then \( L^F_1 = 0.1718 \).

With two domestic firms, the private firm’s profit is part of domestic welfare:

\[ D = \int_0^{\epsilon_1} \frac{1}{2} [r - t(L_1 - x)]^2 dx + \int_{L_1}^{\epsilon_1} \frac{1}{2} [r - t(x - L_1)]^2 dx + \pi_2 \]  

where \( \pi_2 \) is domestic private firm’s profit.

\[ \pi_2 = \int_{L_1}^{\epsilon_1} (c_1 - c_2) [r - t(x - L_1)] dx \]

\[ L_{12} = \frac{6r + 7L_1 - 2\sqrt{9r^2 + 14trL_1 + 7t^2L_1^2 - 14tL_1 + 7t^2 - 14t}}{7t} \]  

To illustrate we again let \( r = 1 \) and \( t = 1/4 \). Solving Eq. (A6) simultaneously:

\[ L_1^F = 0.2804 \]
\[ L_2^F = 0.7490 \]
\[ D = 0.4670 \]

Now we consider the addition of a foreign firm on the left, \( L_0 \), of the public firm. The domestic welfare remains identical to Eq. (A4), which is independent of the foreign firm’s location. Similarly, the welfare maximization and profit maximization conditions generate the same domestic best responses as in Eq. (A6) and the equilibrium:

\[ L_0^F = 0.0950 \]
\[ L_1^F = 0.2804 \]
\[ L_2^F = 0.7490 \]
\[ D = 0.4670 \]

The locations of domestic firms and domestic welfare remain those of the domestic duopoly. The presence of the foreign firm in this case does not change domestic welfare.

Appendix A. Spatial price discrimination with elastic demand

For each point \( x \), there exists an elastic linear demand \( q(x) = r - p(x) \). In this environment, the public firm adopts the price schedule of its own transport cost, \( t \), in order to gain in the price competition. For a public monopoly, the welfare function is:

\[ D = \int_0^{\epsilon_1} \frac{1}{2} [r - t(L^P - x)]^2 dx + \int_{L^P}^{\epsilon_1} \frac{1}{2} [r - t(x - L^P)]^2 dx \]  

(A1)

The welfare maximization generates equilibrium:

\[ L^P = \frac{t}{2} \]
\[ D = \frac{t^2}{24} - \frac{tr}{4} + \frac{r^2}{2} \]  

(A2)

In a foreign mixed duopoly, where there is a public firm and a foreign private firm, both firms will adopt public firm’s transport cost as price schedule. Therefore, the welfare function is identical to Eq. (A1) and is independent of the foreign firm’s location. The foreign firm maximizes its profit

\[ \pi_F^* = \int_0^{\epsilon_1} (\epsilon_F - c_1) t (L^F - x) dx \]

Thus, the public firm location remains unchanged while that of the foreign private firm is

\[ L^F_1 = \frac{7t - 12r + 2\sqrt{7t^2 - 28tr + 36r^2}}{14t} \]  

The intuition is that entry of a foreign firm reduces transport cost and cost reduction is the foreign firm’s profit. This foreign profit is shifted out of the country but leaves domestic welfare unchanged. As an illustration, if \( r = 1 \) and \( t = 1/4 \), then \( L^F_1 = 0.1718 \).

With two domestic firms, the private firm’s profit is part of domestic welfare:

\[ D = \int_0^{\epsilon_1} \frac{1}{2} [r - t(L_1 - x)]^2 dx + \int_{L_1}^{\epsilon_1} \frac{1}{2} [r - t(x - L_1)]^2 dx + \pi_2 \]  

where \( \pi_2 \) is domestic private firm’s profit.

\[ \pi_2 = \int_{L_1}^{\epsilon_1} (c_1 - c_2) [r - t(x - L_1)] dx \]

\[ L_{12} = \frac{6r + 7L_1 - 2\sqrt{9r^2 + 14trL_1 + 7t^2L_1^2 - 14tL_1 + 7t^2 - 14t}}{7t} \]  

To illustrate we again let \( r = 1 \) and \( t = 1/4 \). Solving Eq. (A6) simultaneously:

\[ L_1^F = 0.2804 \]
\[ L_2^F = 0.7490 \]
\[ D = 0.4670 \]

Now we consider the addition of a foreign firm on the left, \( L_0 \), of the public firm. The domestic welfare remains identical to Eq. (A4), which is independent of the foreign firm’s location. Similarly, the welfare maximization and profit maximization conditions generate the same domestic best responses as in Eq. (A6) and the equilibrium:

\[ L_0^F = 0.0950 \]
\[ L_1^F = 0.2804 \]
\[ L_2^F = 0.7490 \]
\[ D = 0.4670 \]

The locations of domestic firms and domestic welfare remain those of the domestic duopoly. The presence of the foreign firm in this case does not change domestic welfare.

Appendix B. Proof of Proposition 7

i.) The payoffs (Table 1) yield inequalities that form necessary and sufficient conditions for three equilibria when the public firm is not next to the foreign firm, \( FL, FFL \) and \( LFL \):

\[ n_{FL} > n_{FL}^d, n_{FL} > n_{FL}^d, D_{FL} > D_{FL} \]
\[ n_{FL} > n_{FL}^d, n_{FL} > n_{FL}^d, D_{FL} > D_{FL} \]
\[ n_{FL} > n_{FL}^d, n_{FL} > n_{FL}^d, D_{FL} > D_{FL} \]
Appendix C. The equilibrium of foreign leader for $N$-firm case

When the public firm is next to the foreign firm, to prevent the extra corner profit loss from the foreign firm, the public firm is always better off by jumping to the left hand side of the foreign leader, $l^0 = \frac{(2m - 1)t^4 + 2}{2m + 3\sqrt{6} - 1}$. Knowing the best response of the public firm, the no-jump conditions further generate the equilibrium location of the foreign leader:

$$I^f = \begin{cases} \frac{3\sqrt{6} - 2}{2m + 3\sqrt{6} - 1} & \text{if } m > 2 \\ 1 & \text{if } m \leq 2 \end{cases}$$

And thus, the domestic welfare and global welfare become:

$$D^f = \begin{cases} r - \frac{(2m + 17)t}{2(2m + 3\sqrt{6} - 1)^2} & \text{if } m > 2 \\ r - \frac{(2m^2 + 9m + 3)t}{12(2m + 1)^2} & \text{if } m \leq 2 \end{cases}$$

$$W^f = \begin{cases} r - \frac{(2m + 21 - 4\sqrt{6})t}{2(2m + 3\sqrt{6} - 1)^2} & \text{if } m > 2 \\ r - \frac{t(2m + 3)(m + 1)}{12(2m + 1)^2} & \text{if } m \leq 2 \end{cases}$$

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